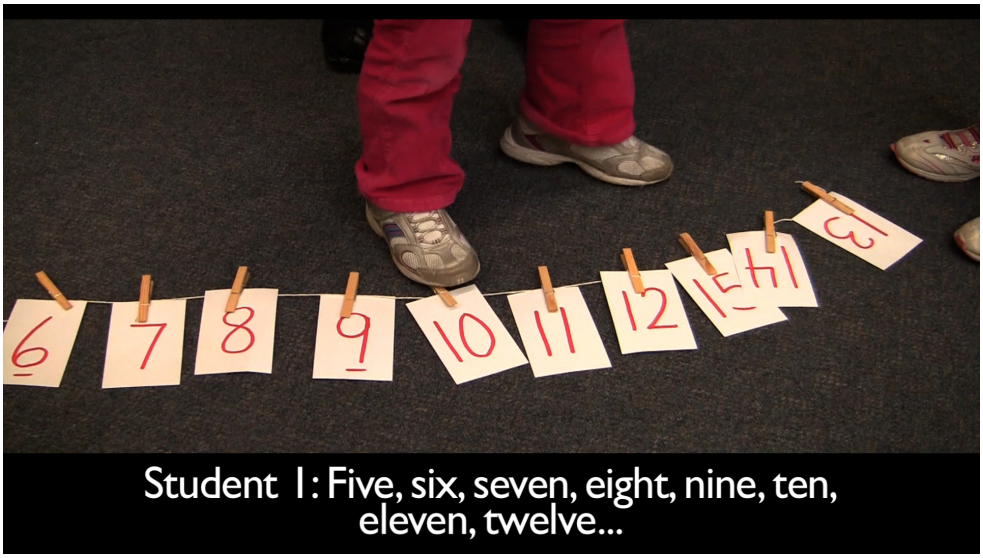
PETERBOROUGH LESSON STUDY TEAM
KPRDSB, Kindergarten to Grade 2
APRIL 2, 2012

Research Question:

How does the number line work as a thinking tool to support students in developing number sense and proportional reasoning?
(Moving beyond ordinality)

How can the number line prepare students for future mathematics (such as learning fractions, multiplicative reasoning)?



Teacher-Researcher Team:

Edmison Heights: Juli McCully, Christina Leeking Hess
RF Downey: Susan Burns, Karen Dillon, Tracey Johnston, Lisa Roy
Numeracy Coach: Teresa Kingston
Trent University: Catherine D. Bruce, Tara Flynn, Diana Chang
Discussant: Janice Mackenzie

KAWARTHA PINE RIDGE DISTRICT SCHOOL BOARD
TRENT UNIVERSITY
Ontario

Table of Contents

"I love that young students are learning how to use the word *INTERVAL*." (Julie)

AGENDA

10:00 to 11:00	Welcome, Introductions, and background provided by the planning team
11:15 to 11:30	Break and Networking and Snack
11:30 to 12:15	Public Lesson
12:15 to 1:30	Debrief: i. Teachers who taught lesson ii. Observers from teacher planning group iii. Comments from guests iv. Discussant

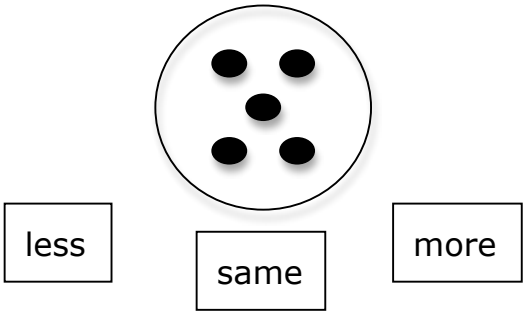
Background Information

Three key sources of information:

1. Task-based interviews

The interview tasks were developed by the team in order to uncover students' early number sense. Six students from each class were interviewed by the teacher with the support of researchers; the interviews were videotaped and analyzed as a group. Our observations and findings led us to want to explore the number line further.

Question 1:
Materials: dot plate with 5 dots, circular counters, card/stickies that read "less", "same", "more").

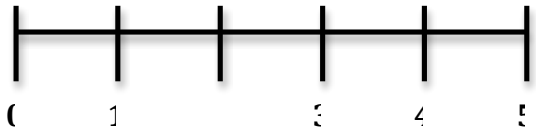


Use the counters to show me the same amount. (place card with word SAME below dot plate)
Now show me less (place card with word LESS to left and below of dot plate)
Now show me more. (place card with word MORE to right and below dot plate)

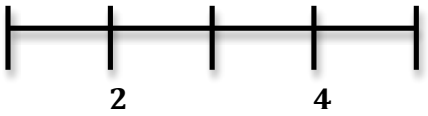
Grade 2 modification: two 5 dot plates and one 3

Question 2. (Materials: number lines on paper, markers)

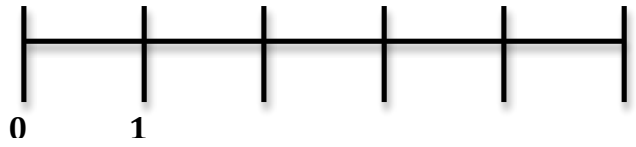
Here is a number line.
What number is missing?
How do you know?



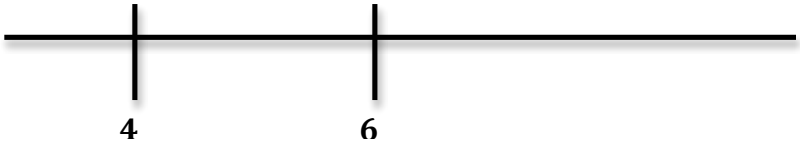
Here is another number line.
Where does the number 1 go?
Show me where you think the number 1 goes.
How do you know?



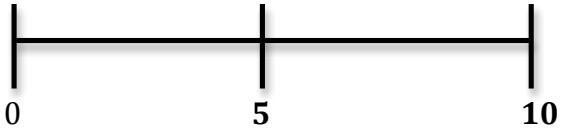
Here is another number line. Where does the number 3 go? How do you know?

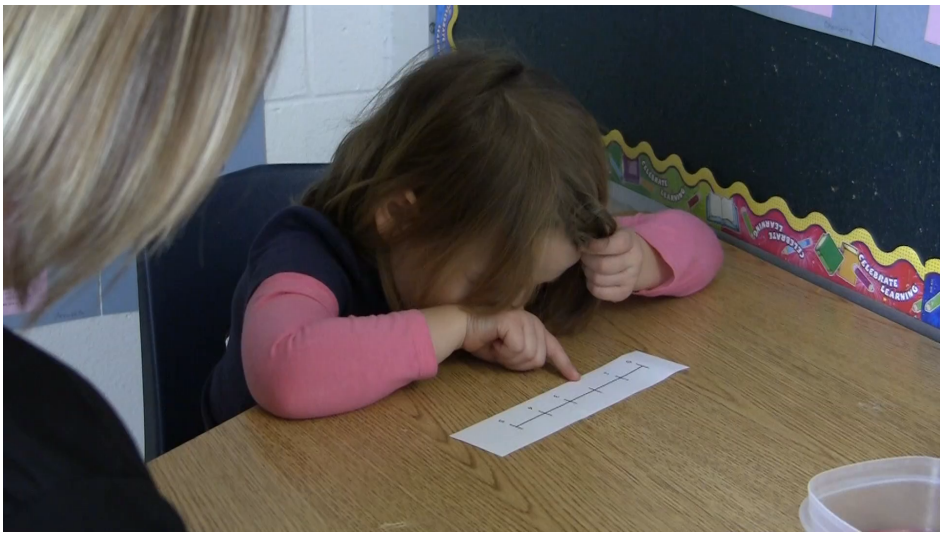


Here is another number line. Where would you put the number 7? How do you know?



Here is another number line. Where does the number 8 go?
Show me with a marker.
How do you know?





Sophie uses the number line as a thinking tool.

Question 3:
(Materials: centicubes, all different colours)

I have some cubes in my hand. I'm going to show them to you and you are going to tell me how many. Don't worry about counting the cubes. Just take a guess. Ready? Because I'm not going to show them for long.
(have 7 in your hand behind your back, show them for 2 seconds)

How many do you think there are? Let's check. [Grade 2 modification: show 11 cubes next]

Was your guess close?
Was it too high, too low?

Question 4:
(Materials: bears, cup of counters)

I'm going to tell you a little story. Here are some bears [show counters]. One day there were 4 bears in the woods, then 3 more came along to play. How many bears are there in total? (give students bear counters to show their thinking)

[7]

If you are going to give each of them an apple, how many would you need?
[remember you have 7 bears]
[mental math]

How do you know?

We need 7 apples, one for each bear. Let's say 5 bears already have apples. Here are some counters, let's pretend these are apples. How many more do you need so every bear gets one apple?
(cup of counters)

[*Show me how many you have already? How many more do you need? How do you know?]*

Question 5:

(Materials: 12 cubes all one colour; one rod has 8, one has 4 – have a few sets ready to give students each time)

Here are 2 sets of cubes. Are they equal/the same? [What else do you notice?]
("Equal means the same" if students don't know the word e*qual.)*

How do you know?

How can you make these two sets equal/the same?

Show me another way? Tell me what you did.

*Show me another way? Tell me what you did? (*Repeat until all ideas are exhausted)

General Observation:
The task-based interviews signaled that young children were able to use the number line intuitively to help them make sense of magnitude, ordinality, cardinality, relative quantity, etc. This got us very excited about maximizing this simple thinking tool for students. We also immediately recognized the potential power of this tool for continued use in the upper grades.

2. Literature Background

Summary Chart Created by the Peterborough Lesson Study Team

		Types of Number Lines (what research?)	Concepts	Teaching Ideas	Because later in life…
Number line	**Earlier**	Missing Values (Saxe, 2010) -Fixed number line -Numbers missing	• Counting and counting on (ordinality) • Number recognition • Sequencing • Proportional reasoning	- Show student a number line with a few numbers missing (oh no! some numbers fell off the number line) - Have students take a stack of number cards that are unordered and place them on a number line so that they are in order	-Life skills -Number knowledge -Ordinality -Use of benchmarks -Proportions and equal intervals of distance
		Board Game (Seigler & Romani, 2008) -Fixed line -Fixed numbers	• Numerical magnitude comparison • Number line estimation • Counting • Numeral identification • Counting on	- Make a board game from 0 to 10 called "Race to 10". Make a spinner with 2 sections (1 and 2). Spin to see if you move 1 or 2 spaces. First person who gets to 10 wins (*use oral counting on*). Consider a variation with Race to 10 and back again. Consider making it so students must land on 10. Consider making a spinner partitioned into 4 equal parts with numbers 1-4.	-Conservation of number -Addition (and subtraction) - Counting on - Understanding magnitude - Prediction skills (how many more spins?) - Number sense skills
	Later	Changing Intervals (Griffin, 2004) -Fixed line -Changing numbers at start and end of line	• Equality • Magnitude • Number knowledge	- Line stays the same but intervals and numbers change Show a line with 0-10: Where does 5 go on this number line? Next change the line to 0-20: Now where does 5 go? Next show number line 0-100: Now where does 5 go? Next change line to -10-+10: Now where does 5 go? What do you notice?	-Money -Fractions -Benchmarks -Conservation -Integers -Less than (related to subtraction) -Measurement
		Open number line / Empty number line (Bobis, 2007) -Fixed line -Changing and relative numbers	• Relative magnitude • Rational numbers • Reasoning	Present a number line. Should we put on the ends? Now let's add another number. -Use whiteboards or magnets/felt -More than/less than Oh no! ALL the numbers fell off this number line. Can you put it back together? (intervals and numbers)	-Fractions -Relative magnitude -Measurement -Benchmarks

3. Our early thinking about the number line:

Students were having success with:

- Counting on a number line
- Ordering numbers on a number line
- Recognizing that intervals were needed on a number line

What we noticed that the the students struggling with:

- Not putting all numbers on the number line but still identifying where a given number would go
- Paying attention to spacing of the intervals (cramming them together, unequal partitioning)
- Where 5 goes on a number line when the end point changes or when the number line itself gets longer or shorter
- Benchmark numbers to support placement of numbers

Misconceptions for some Kindergarten students: "All number lines start at zero" "5 is always in the middle" / Played the up to 10 game. Most of JKs didn't recognize the numbers to 10. (next step: homework version of the game)

Julie: students had trouble finding the half way mark/5 middle / lots of folding.
But working only with numbers to 10. Now let's look at 20 (and NOW what number goes in the middle)

Fixed number line but different lengths. Find the middle. ("it needs to be in the middle" – very obvious). Then the intervals were discussed in detail, talked about how there aren't different sized intervals.
Full number line to ¾ number line – put the 5 directly below the 5 of the previous number line.

Cathy talked about conservation of length (wondering how that effects length)

Lined all 5's up: what do you notice that is different in these number lines?
0's all the same, 5's all the same.
"Wait a second, the space changes but the numbers don't"
Use the 100's chart to find the middle number. Found 50 on a hundred chart to find the middle.

Conception:
Spacing matters!

Exploratory Lessons

Lesson Sequencing

a. The anatomy of the number line: Oh no, some numbers fell off!
b. Unusual number lines (e.g., a number line that begins at 5 and ends at 15)
c. Finding 5 on different number lines (0 to 10, 0 to 15, 0 to 20, 0 to 100)
d. Using the benchmark of one half to help find intervals and related numbers (what number is in the middle?)
e. Investigating hops (spacing of intervals and the physical-ness of the spacing)

What we have learned so far:

What we have learned so far about: Using the Number Line as a Tool for Mathematics	**Example from our Lesson Study Research**
Working with number lines is much more than just learning about the number line: One little part of the curriculum can affect many other parts of the curriculum	Number line addresses so many expectations across the math strands and curriculum; Not just learning the numbers Number lines help us teach ordinality but also cardinality, and proportional reasoning (see group chart) Negative numbers and zero are interesting to students!! Potential use of number line for operations
The number line is so powerful that we would teach and plan mathematics differently.	Change whole start of the year – working with number lines to establish some key basics with students
Some students have misconceptions about number lines and challenges	For some of our Kindergarten students: "All number lines start at zero or start at 1" "5 is always in the middle" / Played the up to 10 game. Many of JKs didn't recognize the numbers to 10.
Students are able to make important generalizations using number lines.	Spacing matters! (and is affected by the numbers on the line – the start and end points)
Some students have a sense of quantity and magnitude (e.g., that 8 is between 5 and 10) without necessarily knowing what numbers go where (or what numbers are missing) on a number line	Clinical interviews: For students who had very low number recognition, they had trouble finding the missing numbers on the line, BUT students had the idea that quantity of 8 was between 5 and 10 but couldn't find the number
It takes time to build an understanding of intervals: Intervals – it is important for students who are new to the number line	Students don't seem to have much experience with a number line prior to school. They need to have explicit instruction to understand how intervals work and the anatomy of the number line Spacing, etc. on the number line important including using Velcro, marked intervals and objects that fill the interval for equal spacing. Students need exposure to the number line. Students then make connections on their own (to other activities)
Recognition, then Recall: Students may have a rote sense of numbers but may not recognize the symbols.	Student can count to 10 but may not be able to name what the name of the number is when they see the symbol.
Students need opportunities to use math language	More, less, middle, half, interval, spacing, understanding what space means, small, growing, increasing, number line, amount
The line is fixed but the numbers and the intervals can be made flexible	Students are thinking about how there can be numbers before 1, such as 0 and before 0 such as negative numbers. Young children are discussing this! Flexible lines don't work well (no string… ever!)
Look at all the numbers at once in a line (an order, and always visible)	Reinforces number sense in relationship to one another.
Number line digs into a greater understanding of number (and math processes: Problem solving, Communication, Reasoning, Relative-ness)	Helps to set students up for success - see chart
Counting on: The Race to 10 Game really works (continue to play the game)	Playing the Race to 10 Game: Connects to other uses of math and number sense Only 4 small spoons left. Oh but here I just found 2. So now there's 6! 4 and 2 more.
Need to spend more time on number lines to deepen student understanding, and play with the number line as a learning tool.	Question: Will this use of the number line now have an effect for later learning? (still to come – but even just thinking about measurement and rulers and how they connect to the number line) The tasks that we are using are much more rich, so we can spend more time on it, and deepen student understanding
The tool itself is critically important: Each choice is so important	Strips of paper, versus string, some items are distracting, some are helpful.
AS a teacher – thinking about asset model (what do the	Pleasantly shocked each time we do a task; students come to school rote counting

students HAVE)	and some sense of ordinality – take advantage of this and build on it

Public Lesson

Goal of public lesson:

a. The distance between two adjacent whole numbers is always the same on a number line: The intervals need to be equally spaced because it helps to build understanding of whole numbers and what might be between two whole numbers
b. When a number line changes length, but the start and end numbers stay the same, then the spacing (or intervals) change in distance (the partitioning of intervals stretches or condenses)

Specific Learning Goal:

By the end of this lesson, we hope that students will understand that the length of a number line (where 0 is on one end, and 10 is on the other end) affects the spacing of intervals. We also hope that students will compare number lines and notice that the location of 5 is relative to the length of the line.

Materials:

- Two incorrect number lines
- Practice hops number lines (one side with intervals, the other no intervals)
- Custom cut papers for each student with two number lines (one is the full length of the paper, while the other is half the length) – names on the sheets in advance both top and bottom, and numbered in advance
- Sticky tabs
- Two metre number line
- Pencils
- Scissors

Activation of student thinking

- Present "a wrong" number line: 0-10 (all intervals jammed 0-9, then big space, then 10) (sentence strips with tabs)
- "I just made this number line. What do you think about my number line?"
- Okay, so we need to space everything out. Here's another number line I made. What do you think about this number line?
- Present a second "wrong" number line: 0-10 with most spaces accurate but ONE not (double the space between 2 and 3)
- Give students the opportunity to revise the second number line whole group

Development of ideas

Material: One long number line from 0 to 10; intervals on one side and not the other
Whole Group

1. Use your finger to jump from 0 to 10 and show all the hops along the way. (Let's practice a hop in the air) (What do you notice?)
2. Now turn it over and Jump with your finger from 0 to 10 showing all the hops (on the empty number line) (How was jumping different on the first number line, compared to the second number line?)
3. Distribute think sheet with 2 number lines, one short and one long (special custom cut) for each student (make extra number lines available); Have students work in pairs: "Use your finger to practice jumps from 0 to 10; When you are done, come and get a pencil,

and then mark your jumps with a pencil on the two number lines. You can use multiple sheets until you are happy with your jumps. ENCOURAGE STUDENTS TO THINK OUT LOUD AS MUCH AS POSSIBLE.

4. Explain to your partner: Which number line shows the best hops? Why? Have a discussion with your partner.
5. Have students come back to carpet with their "best number lines": Where would 5 go on your number lines? Here are two tabs, mark 5 on each number line with a tab. (cut the number lines)
6. Whole group discussion: How did you find 5? How do you know you are right?
7. Take up different solutions: Start with longer number lines: Oh we have two different ideas about where 5 goes. Tell us what you were thinking when you put 5 here? (to each pair). What do other people think? Where would 5 go? (Be sure the number lines are lined up for comparisons)
8. Compare two number lines of different length: Oh this interesting. We have one number line where 5 goes here, and another number where 5 goes here. Why do you think that is? (Be sure the number lines are lined up for comparisons)

Consolidation of student thinking (alternate ending – if students are still able to continue)

Show students a 2 metre long number line (0 to 10) and ask where does 5 go now?
(sticky for where it should go) (thumbs up, thumbs down) If you think it should be in a different place, tell us about your idea.
Have a discussion.

Observation Guides

1. Student Communication (gestures and actions as well as language)
2. Use of number line as a thinking tool (to find the intervals, to find 5)
3. Self-correction strategies (multiple attempts; revising thinking whole group)
4. Lesson synthesis (functioning of the lesson and flow, and connections between the parts of the lesson)
5. Planning team will be scribing for pairs of students

Date: April 2, 2012
Name:

Observation Guide 1:
Student Communication

How do students use **math language** during the tasks to express their mathematical thinking?

- to reason
- specific language: middle, halfway, interval, more, less, greater than, less than, equal

How do students use **gestures and actions** to express their mathematical thinking to a peer, the group etc?

Date: April 2, 2012
Name:

Observation Guide 2:
Use of the Number Line as a Thinking Tool

To find intervals:

What strategies are students using to determine the intervals?

- i.e., gestures and actions
    - folding finger moves/hand gestures use of tools?

To find 5: What strategies are students using to find 5 on the number line?

• Student uses ordinal counting strategy	
• Student uses *visual* benchmark strategy (partitioning strategies, finding halfway)	
• Student uses *number* benchmarking strategy (eg., "5 is half of 10")	
• Student uses another strategy (explain)	

Date: April 2, 2012
Name:

Observation Guide 3:
Self-correction strategies

Number of attempts/sheets used by students each time	Change in gestures/movements/use of tools
Language (eg. "that doesn't work" – talking it through)	Did finger jumping help? How many attempts did students make before committing to pencil each time?
Evidence of revised thinking in the whole group (eg. "I did think this and now I think this…)	Other

Date: April 2, 2012
Name:

Observation Guide 4: Lesson Synthesis

General notes on **Activation**:

General notes on **Development**:

General notes on **Consolidation**:

Reflection:

What aspects of activation came Out in other areas of the lesson?	What aspects of activation and development came out in the consolidation?